FRICTION AND HEAT TRANSFER IN THE TURBULENT BOUNDARY LAYER OF ROTATING SYSTEMS

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Rotating a flow or a solid surface is an effective method for intensifying or suppressing turbulent transfer. Rotation is widely employed in modern power plants. Research in this direction is becoming increasingly broader.

Swirling of flow or rotation of the surface can change the friction and heat and mass transfer primarily by two mechanisms [1-3]. First, an increase in the velocity vectorat the outer boundary of the boundary layer due to rotation of the flow or the surface increases, by increasing the turbulent diffusion coefficients, the gradients of the average velocity and correspondingly the tangential stresses and heat fluxes. Second, in rotating flows the flow and heat transfer are affected by the effect of the centrifugal and Coriolis body forces acting on the turbulent structure of the flow. It is well-known that body forces can both intensify and suppress turbulent diffusion [1-3]. Buoyancy effects due to gradients of the gas density, which arise in nonisothermal and compressible flows, should also be included among body forces.

Especially complicated flows are observed when all of the factors indicated above are present simultaneously, as happens, for example, in real flows on the rotating blades of gas turbines. The solution of the problem of friction and heat transfer for such complicated gas-dynamic conditions is an important problem.

A method of calculating the effect of centrifugal forces on turbulent heat and mass transfer in the boundary layer of swirling and curvilinear flows has been developed in a number of works [1, 4, 5]. It is based on taking into account the changes produced in the velocity pulsations normal to the surface by centrifugal forces. As a result, expressions were obtained for the tangential stresses and heat fluxes. The method is similar to the analysis made in [6]. But, in contrast to [6], the application of the law of conservation of angular momentum for a separate pulsational particle of fluid makes it possible to calculate the friction and heat transfer without introducing additional empirical constants. Comparing the two methods showed that, in spite of a number of assumptions which were made, the working relations obtained for swirling and curvilinear flows agree satisfactorily with the experimental data [1, 4, 5].

Heat transfer and friction at rotating curvilinear walls, in the general case in swirling flows, can also be analyzed from this viewpoint, though the problem becomes significantly more complicated.

We underscore especially the fact that besides the effects of curvature and rotation, which act on the turbulence field, in such systems significant restructuring of the average velocity fields occurs as a result of the appearance of secondary and detached flows. This in turn can give rise to a change in heat transfer and friction than is produced by centrifugal and Coriolis forces. This problem is of interest in itself, and we shall not consider it in this paper; the velocity distribution outside the boundary layer is assumed to be known and is found, to a first approximation, from the solution of the nonviscous flow.

1. Formulation of the Problem. Choice of Coordinate System. Derivation of Equations of Motion of a Turbulent Particle of Fluid

We shall consider the problem of the effect of centrifugal and Coriolis forces on turbulent velocity pulsations in the most general case of nonisothermal flow over a rotating surface. A schematic diagram of the flow is shown in Fig. 1a. The curvilinear surface N rotates relative to the OO' axis with constant angular velocity Ω_0. A gas flow is directed at this surface. The velocity u_0 of the gas flow outside the boundary layer is given by the expression

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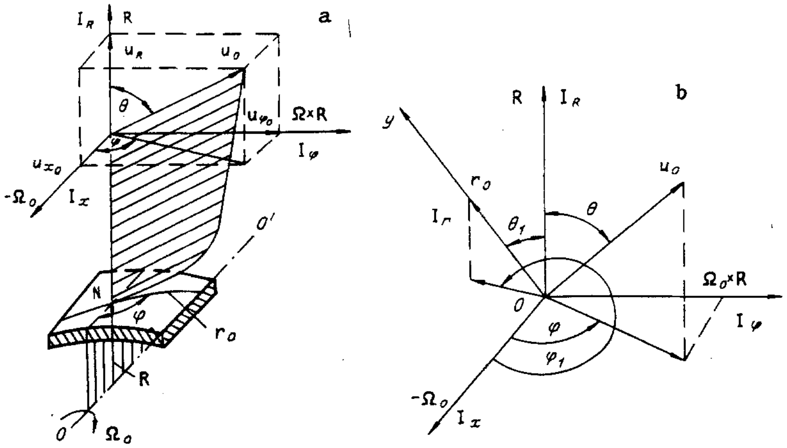

Fig. 1

$$\mathbf{u}_0 = u_0(\mathbf{I}_x \sin\theta \cdot \cos\varphi + \mathbf{I}_\varphi \sin\theta \cdot \sin\varphi + \mathbf{I}_R \cos\theta), \tag{1.1}$$

where $\mathbf{I}_x$, $\mathbf{I}_\varphi$, and $\mathbf{I}_R$ are unit vectors in the direction of the x, φ, and R axes. The vectors $-\mathbf{\Omega}_0$, $\mathbf{\Omega}_0 \times \mathbf{R}$, and $\mathbf{R}$ are oriented in the direction of these unit vectors.

The streamlines at the suface are not oriented in the same direction as the rotation axis and make an angle φ with it. The curvature of the streamlines characterized by the radius vector $\mathbf{r}_0$, whose direction is determined by the angle θ_1 and φ_1 (Fig. 1b):

$$\mathbf{r}_0 = r_0(\mathbf{I}_x \sin\theta_1 \cdot \cos\varphi_1 + \mathbf{I}_\varphi \sin\varphi_1 \cdot \sin\theta_1 + \mathbf{I}_R \cos\theta_1). \tag{1.2}$$

The angles θ, φ and θ_1, φ_1 are related with one another by the condition that the vectors $\mathbf{u}_0$ and $\mathbf{r}_0$ are orthogonal to one another:

$$\mathbf{u}_0 \cdot \mathbf{r}_0 = 0. \tag{1.3}$$

Substituting Eqs. (1.1) and (1.2) into Eq. (1.3), we obtain

$$\cos\varphi_1 = -\mathrm{ctg}\,\theta \cdot \mathrm{ctg}\,\theta_1 \cdot \cos\varphi \pm \sqrt{\sin^2\varphi(1 - \mathrm{ctg}^2\theta \cdot \mathrm{ctg}^2\theta_1)}.$$

Following [1, 4, 5], we write the equation of motion for a particle of fluid in the coordinate system moving with the particle, so that only the pulsational motion in the direction of the vector $\mathbf{r}_0$ remains. As previously, the viscous effects of the interaction of a single particle of fluid with the surrounding medium are neglected. The vector equation of motion of the turbulent particle is as follows:

$$\rho_s d\mathbf{u}_s'/dt = \rho_s\mathbf{f} - \nabla p \tag{1.4}$$

where ρ_s and $\mathbf{u}_s$ are the density and the velocity vector of the fluid particle and p is the average pressure. The quantity $\mathbf{f}$ is the sum of all inertial forces acting on the particle in pulsational motion. In the general case, when the wall and the flow rotate, the motion of the turbulent particle can be represented as a superposition of two rotations, determined by the rotation of the surface and the rotation of the gas flow relative to the surface. The angular velocity of the fluid particle is equal to $\mathbf{\Omega}_{0s}$, in the first case and ω_s in the second case. In such a coordinate system rotating with the fluid particle, only the motion of the particle normal to the surface of the body remains. Then the total body force acting on the fluid particle can be written in the form $\mathbf{f} = \mathbf{f}_0 + \mathbf{f}_1 + \mathbf{f}_2$, where $\mathbf{f}_0$ is the mass force giving rise to velocity pulsation in a flat boundary layer in the absence of rotation. The mass forces $\mathbf{f}_1$ and $\mathbf{f}_2$ due to rotation of the system and the flow, repsectively, are determined by the expressions

$$\mathbf{f}_1 = -\left[2\boldsymbol{\Omega}_{0s}\times\mathbf{u}_s + \boldsymbol{\Omega}_{0s}\times(\boldsymbol{\Omega}_{0s}\times\mathbf{R}) + \frac{d\boldsymbol{\Omega}_{0s}}{dt}\times\mathbf{R}\right]; \tag{1.5}$$

$$\mathbf{f}_2 = -\left[2\boldsymbol{\omega}_s\times\mathbf{u}_r' + \boldsymbol{\omega}_s\times(\boldsymbol{\omega}_s\times\mathbf{r}_0) + \frac{d\boldsymbol{\omega}_s}{dt}\times\mathbf{r}_0\right]. \tag{1.6}$$

The first term on the right-hand side of the relations (1.5) and (1.6) is the Coriolis force, the second term is the centrifugal force, and the third term is due to the nonstationary character of the rotation of the system or the flow. In what follows, for simplicity, we confine our attention to stationary rotating flows.

As done previously in [1, 4, 5], we assume that the body forces affect predominantly the component of velocity pulsations that is normal to the surface. Then we write the projection of Eq. (1.4) on a direction normal to the surface (in so doing, we assume that $\mathbf{f}_2\cdot\mathbf{I}_r = \omega_s^2 r = u_s^2/r$, where u_s is the velocity of the fluid particle along the surface over which the fluid flows) as follows:

$$\frac{1}{2}\frac{du_r'^2}{dr} = f_0 - \left[2\boldsymbol{\Omega}_{0s}\times\mathbf{u}_s + \boldsymbol{\Omega}_{0s}\times(\boldsymbol{\Omega}_{0s}\times\mathbf{R}) - \right. \tag{1.7}$$

$$\left. -\frac{\rho}{\rho_s}\Big((2\boldsymbol{\Omega}_0\times\mathbf{u}) + \boldsymbol{\Omega}_0\times(\boldsymbol{\Omega}_0\times\mathbf{R})\Big)\right]\cdot\mathbf{I}_r + \left(\frac{u_s^2}{r} - \frac{u^2\rho}{r\rho_s}\right).$$

The relation (1.7) was written taking into account the fact that the projection of the pressure gradient on a direction normal to the surface has the form

$$\nabla p\cdot\mathbf{I}_r \equiv \frac{\partial p}{\partial r} = \frac{\rho u^2}{r} - \rho\left[2\boldsymbol{\Omega}_0\times\mathbf{u} + \boldsymbol{\Omega}_0\times(\boldsymbol{\Omega}_0\times\mathbf{R})\right]\cdot\mathbf{I}_r.$$

In particular, if we set $\Omega_0 = \Omega_{0s} = 0$, in Eq. (7), then we obtain the equation employed in [1, 4, 5] for curvilinear and swirling flows. We assume that during the pulsational motion of the fluid particle the circulation of the particle $u_s r = (ur)_{r+\Delta r}$ ($r = c_1 y$, where y is the distance from the wall; $c_1 = 1$ for flow over a concave surface and $c_1 = -1$ for a convex surface). By analogy to [4], expanding the circulation and density in a series in y and neglecting the quadriatic and higher-order terms, we have

$$u_s r \approx ur - \frac{\partial ur}{\partial y}y,\ (u_s r)^2 \approx (ur)^2 - \frac{\partial (ur)^2}{\partial y}y,$$

$$\frac{\rho}{\rho_s} \approx 1\Big/\left(1 - 1/\rho\frac{\partial\rho}{\partial y}y\right) \approx 1 + \frac{1}{\rho}\frac{\partial\rho}{\partial y}y.$$

Since $\Omega_{0s}R^2 = \Omega_0(R+\Delta R)^2$, where $\Delta R = c_2 y\cos\theta_1$, we obtain

$$\Omega_{0s} = \Omega_0(1 + 2c_2 y/R\cos\theta_1).$$

The coefficient $c_2 = 1$ or -1, if the flow over the surface is on the same side of the surface as the axis of rotation or on the opposite side, respectively. Using the relations obtained, Eq. (1.7) can be put into the form

$$\frac{1}{2}\frac{\partial u_y'^2}{\partial y} = c_1(f_0 + ky). \tag{1.8}$$

Here

$$k = -2(\cos\theta\cdot\sin\theta_1\cdot\sin\varphi_1 - \cos\theta_1\cdot\sin\theta\cdot\sin\varphi)\times \tag{1.9}$$

$$\times\left(-\frac{1}{r}\frac{\partial ur}{\partial y}-\frac{u}{\rho}\frac{\partial\rho}{\partial y}+\frac{2c_2 u\cos\theta_1}{R}\right)\Omega_0+$$
$$+\cos\theta_1\left(-\frac{1}{\rho}\frac{\partial\rho}{\partial y}+\frac{4c_2\cos\theta_1}{R}\right)\Omega_0^2 R-\left(\frac{2u}{r^2}\frac{\partial ur}{\partial y}+\frac{1}{\rho}\frac{\partial\rho}{\partial y}\frac{u^2}{r}\right).$$

It is convenient to represent the function k as

$$k=k_0 k_1, \tag{1.10}$$

where the parameter $k_0=-\frac{2u}{r^2}\frac{\partial ur}{\partial y}$ reflects the effect of body forces in curvilinear flow but without rotation of the entire system; k_1 is a function that describes the contribution of the rotation of the system as well as buoyancy effects. From the relations (1.9) and (1.10) we obtain

$$\begin{aligned}k_1=&-2(\cos\theta\cdot\sin\theta_1\cdot\sin\varphi_1-\cos\theta_1\cdot\sin\theta\cdot\sin\varphi)\times\\&\times\left(\frac{r\Omega_0}{2u}+\frac{\frac{\Omega_0}{\rho}\frac{\partial\rho}{\partial y}}{\frac{2}{r^2}\frac{\partial ur}{\partial y}}-\frac{2c_2\Omega_0\cos\theta_1}{\frac{2R}{r^2}\frac{\partial ur}{\partial y}}\right)+\\&+\cos\theta_1\left(\frac{\frac{1}{\rho}\frac{\partial\rho}{\partial y}}{\frac{2u}{r^2}\frac{\partial ur}{\partial r}}-\frac{2c_2\cos\theta_1}{\frac{uR}{r^2}\frac{\partial ur}{\partial y}}\right)\Omega_0^2 R+\left(1+\frac{\frac{u}{\rho}\frac{\partial\rho}{\partial y}}{\frac{2\partial ur}{r\,\partial y}}\right).\end{aligned} \tag{1.11}$$

2. Laws of Heat and Mass Transfer and Friction for a Turbulent Boundary Layer in Rotating Systems

We now find an expression for the normal component of the pulsational velocity. For this integrate Eq. (1.8), assuming that k is constant to a first approximation:

$$u_y'=u_{y0}'F. \tag{2.1}$$

Here u'_{y0} is the pulsational velocity in the standard boundary layer (flat nonrotating plate and no swirling of the flow).

Under conditions when turbulence is intensified by centrifugal and Coriolis forces $c_1k_1 > 0$), by analogy to flow at a concave wall [4], the function F is given by

$$F=\left[1-\left(\frac{y}{l}\right)^2\frac{2c_1k_0k_1}{(\partial u/\partial y)^2}\right]^{1/2}. \tag{2.2}$$

We describe the suppression of turbulence by body forces with the help of an approximating expression of the form [4]

$$F=\left[1+\left(\frac{y}{l}\right)^2\frac{2c_1k_0k_1}{(\partial u/\partial y)^2}\right]^{-1/2}\quad(c_1k_1<0). \tag{2.3}$$

Using the relations (1.11) and (2.1)-(2.3), under the assumption that the field of body forces has no effect on the longitudinal pulsations, and the cross-correlation coefficient remains the same as for nonrotating flow, we obtain the following expressions for the Reynolds stresses and turbulent heat fluxes:

$$\overline{u'_x u'_y} = \overline{u'_{0x} u'_{0y}}\, F; \tag{2.4}$$

$$\overline{T' u'_y} = \overline{T'_0 u'_{0y}}\, F. \tag{2.5}$$

The tangential stress and heat flux at the wall can be calculated, using Eqs. (2.4) and (2.5), just as is done in [1, 4, 5]. For this k_1 must be averaged over the thickness of the boundary layer. Using power-law profiles of the dimensionless velocity and temperature $u/u_0 = (y/\delta)^n$, and $(T - T_{st})/(T_0 - T_{st}) = (y/\delta_T)^n$, respectively, and the equation of state of an ideal gas $\rho/\rho_0 = T_0/T$, we obtain from Eq. (1.11)

$$\begin{aligned} k_1 = N^2 \frac{r_0}{R} \cos\theta_1 \left[\frac{(\psi - 1)(n+1)(2n+1)}{2(1+n+\psi n)} - \frac{4c_2\delta\cos\theta_1}{R}\right] - \\ - \frac{Nr_0}{R}(\cos\theta \cdot \sin\theta_1 \cdot \sin\varphi_1 - \cos\theta_1 \cdot \sin\theta \cdot \sin\varphi) \times \\ \times \left[n + 1 + \frac{(\psi-1)(n+1)}{1+n\psi} - \frac{2c_2\delta\cos\theta_1}{R}\right] + \left[1 + \frac{\psi - 1}{2(1+n\psi)}\right] \end{aligned} \tag{2.6}$$

where $N = \Omega_0 R/u_0$ is the inverse Rossby number and $\psi = T_{st}/T_0$ is the nonisothermality factor. Relations similar to Eqs. (2.4) and (2.5) were derived in [5] for the heat and mass transfer functions in the form of superpositions for a nonrotating system ($\bar{k}_1 = 1$)

$$(c_f/c_{f_0})_{\mathrm{Re}^{**}} = (\mathrm{St}/\mathrm{St}_0)_{\mathrm{Re}_T^{**}} = \Psi_T \Psi_\varphi \Psi_c.$$

Here $\Psi_T = [2/(\sqrt{\psi} + 1)]^2$ is the nonisothermality function; the function $\Psi_\varphi = 1/\cos\varphi^{0,75}$ takes into account the effect of an increase in the velocity gradients owing to swirling of the flow on heat transfer and friction; and the function ψ_c takes into account the effect of centrifugal and Coriolis forces.

It is obvious that for a rotating system Ψ_c must be determined using Eq. (2.6). Then the following expressions will be the analogs of the formulas, derived in [5] for the effect of the curvature of the streamlines on turbulence, for the general case of a rotating curvilinear surface:

$$\Psi_c = \left(1 + 1{,}8 \cdot 10^3 \frac{\delta^{**} c_1 \bar{k}_1}{r_0}\right)^{0,162} \quad (c_1\bar{k}_1 > 0); \tag{2.7}$$

$$\Psi_c = \left(1 - 2{,}2 \cdot 10^3 \frac{\delta^{**} c_1 \bar{k}_1}{r_0}\right)^{-0,115} \quad (c_1\bar{k}_1 < 0). \tag{2.8}$$

Thus the formulas (2.7) and (2.8) make it possible to estimate the change in the surface friction and heat transfer owing to the effect of centrifugal and Coriolis forces and buoyancy effects on the structure of turbulent diffusion. The parameter characterizing the effect of these factors is $\bar{k}_1$, described by the expression (2.6), which in turn depends in a complicated fashion on the intensity N of rotation, the radius of curvature r_0 of the surface, the thickness δ of the boundary layer, the isothermality factor ψ, the degree n of filling in of the profile, etc. For this reason, it is best to consider first a number of simple, particular cases of rotating flows or systems.

3. Particular Cases of Rotating Flows

Swirled Flow in a Stationary Channel. Since $r_0 = R_c/\sin^2\varphi$ (R_c is the radius of the channel axis and φ the velocity vector at the boundary of the boundary layer), $c_1 = 1$, and $N = 0$, we obtain a working formula for the coefficient of friction and heat and mass transfer

$$\Psi_c = \left\{1 + 1{,}8 \cdot 10^3 \frac{\delta^{**}}{R_c} \sin\varphi^2 \left[1 + \frac{\psi - 1}{2(1 + n\psi)}\right]\right\}^{0{,}162}.$$

To a first approximation, the degree of filling in of the velocity profile can be assumed to be the same as in a standard flow: $n = n_0 = 1/7$. This relation was obtained in [5] and it is in good agreement with the experimental data on heat and mass transfer in swirled flows under different conditions: quasi-isothermal, significantly nonisothermal, and with different thicknesses of the dynamic and diffusion layers ($Sc \neq 1$).

In the case of swirled flow in a stationary pipe the centrifugal forces intensify heat exchange with the pipe wall. An increase in the velocity vector at the outer boundary of the boundary layer (effect of the geometric factor) also results in intensification of heat transfer.

Flow in a Pipe Rotating along Its Axis. For this case $\mathrm{tg}\,\varphi = N$, $r_0 = \bar{R}/\sin^2\varphi$, $c_1 = 1$, $c_2 = 1$, $\theta = 90°$, $\theta_1 = 0$, $R = R_c$. From Eqs. (2.7) and (2.8) we have

$$\Psi_c = \left(1 - 2{,}2 \cdot 10^3 \frac{\delta^{**}}{R} A\right)^{-0{,}115},$$

if the complex

$$A \equiv N^2 \left(\frac{(\psi - 1)(n + 1)(2n + 1)}{2(1 + n + \psi n)} - \frac{4\delta}{R_c}\right) +$$

$$+ N \sin\varphi \left(n + 1 + \frac{(\psi - 1)(n + 1)}{1 + n\psi} - \frac{2\delta}{R}\right) + \sin^2\varphi \left(1 + \frac{\psi - 1}{2(1 + n\psi)}\right) < 0;$$

$$\Psi_c = \left\{1 + 1{,}8 \cdot 10^3 \frac{\delta^{**}}{R} A\right\}^{0{,}162}, \quad \text{if} \quad A > 0.$$

Thus, depending on the direction of the heat flux, the rotation of the pipe can both intensify and suppress turbulent diffusion. Under close to isothermal conditions ($\psi \approx 1$), the effect of body forces in a rotating pipe is described by the formula

$$\Psi_c = \left\{1 + 2{,}2 \cdot 10^3 \frac{\delta^{**}}{R} \left[\frac{4\delta N^2}{R} + N \sin\varphi \left(\frac{2\delta}{R} - n - 1\right) - \sin^2\varphi\right]\right\}^{-0{,}115}. \tag{3.1}$$

It follows from Eq. (3.1) that centrifugal and Coriolis forces suppress turbulent heat and momentum transfer at the wall. At the same time, when the velocity vector increases as a result of rotation of the pipe relative to the flow, heat emission and friction are intensified. Then the total coefficient of heat and mass transfer can be both greater and less than for flow in a stationary pipe. In the initial sections of the channel, when the boundary layer is thin and the flow lags significantly behind the rotation of the pipe, the intensification of turbulent transfer owing to an increase in the velocity gradients can exceed the effects of suppression of turbulence by body forces. Farther downstream the rotation of the flow approaches rigid body rotation, intensification effects become small, and the stabilizing effect of rotation on friction and heat transfer will predominate.

Figure 2 shows the decrease in the relative friction and heat transfer functions for flow in a rotating pipe. The points 1 are the experimental data, obtained by Levy and White and analyzed by V. K. Shchukin [7], for the hydraulic resistance in rotating pipes; the points 2 refer to experiments on friction [8]; the points 3 are the data on heat transfer [9]; and, the line represents the calculation according to the formula (3.1) for $\delta/R = 1$. It is obvious that the agreement between the calculation and experiment is only qualitative. The quantitative disagreement could be due to errors in the theoretical analysis as well as incorrectness in comparing the results of theory and experiment. Thus the ratio δ/R in the experiments can be less than unity, and vibrations of pipes and formation of Taylor vortices have a strong effect on the flow.

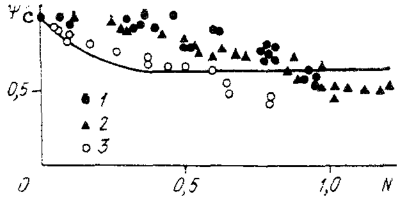

Fig. 2

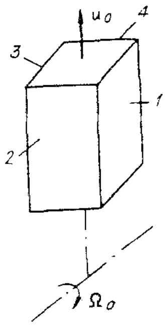

Fig. 3

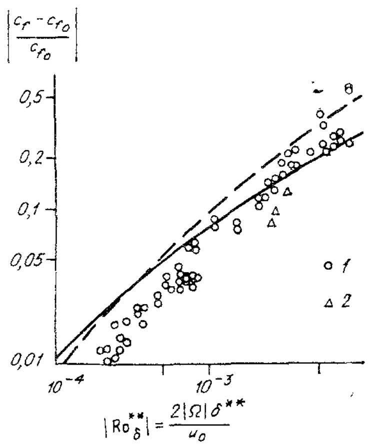

Fig. 4

Rotation of a Rectilinear Channel with the Rotation Axis Perpendicular to the Pipe Axis (Fig. 3). Such flows occur in radial channels of rotating elements of power machines, flat blades, etc. We write down for each of the four walls of the channel the values of the parameters for gas flow entering from the rotation axis.

Wall 1 (rarefaction side): $r_0 \to \infty$, $c_1 = -1$, $\theta_1 = 90°$, $\varphi_1 = 270°$, $\theta = 0$,

$$\Psi_c = \left[1 + 2{,}2 \cdot 10^3 \frac{\delta^{**}N}{R}(1+n)\right]^{-0{,}115}. \tag{3.2}$$

On this wall turbulence is suppressed.

Wall 2 (end wall): $r_0 \to \infty$, $c_1 = -1$, $\theta_1 = 90°$, $\varphi_1 = 180°$, $\theta = 0$, $\Psi_c = 1$. Body forces have no effect on turbulence on this surface.

Wall 3 (pressure side): $r_0 \to \infty$, $c_1 = -1$, $\theta_1 = 90°$, $\varphi_1 = 90°$, $\theta = 0$,

$$\Psi_c = \left[1 + 1{,}8 \cdot 10^3 \frac{\delta^{**}N}{R}(1+n)\right]^{0{,}162}. \tag{3.3}$$

Intensification of turbulence is observed on this surface.

Wall 4 (end wall): $r_0 \to \infty$, $c_1 = -1$, $\theta_1 = 90°$, $\varphi_1 = 0$, $\theta = 0$, $\Psi_c = 1$. Here, as on wall 2, body forces have no effect.

If the flow in the channel is directed in the opposite direction, i.e., from the periphery toward the rotation axis (θ = 90°), the formula (3.3) is valid for the wall 1 and turbulent transfer will be intensified, while at the wall 3 turbulent transfer will be suppressed (formula (3.2)).

Such flow is analyzed in detail in [10]. The case of flow away from the axis is considered and experimental results showing only the effect of rotation on the turbulent transport of momentum are presented. It was established that turbulent transfer is suppressed on the rarefaction side and intensified on the pressure side (points 1 and 2, respectively, in Fig. 4). It is interesting to note that turbulence stabilization and destabilization effects are very similar and can be described by the same relations. Calculations with the formula (3.2) (solid line) and (3.3) (dashed line) are plotted here also. One can see that the calculation agrees with experiment both qualitatively and quantitatively, and the theoretical dependences for the pressure and rarefaction sides are virtually identical.

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